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Even and Odd q-Coherent States in a Finite-Dimensional Basis and Their Squeezing Properties

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The even and odd coherent states of a deformed harmonic oscillator in a finite s-dimensional Hilbert space are studied. It is shown that both for s even and s odd, the even q-coherent states exhibit quadrature and amplitude-squared squeezing, while the odd q-coherent states show an antibunching effect and amplitude-squared squeezing.

1. INTRODUCTION

Of late, q-coherent states have attracted a great deal of attention because of their possible applications in various branches of physics and mathematical physics (Biedenharn, 1989; Macfarlane, 1989; Sun and Fu, 1989; Ng, 1990; Haruo and Aizawa, 1990; Gray and Nelson, 1990; Bracken *et al.*, 1991; Quesne, 1991; Yu, 1992; Chiu *et al.*, 1992; Zhedanov, 1993; Chang, 1992a, b). In particular, the quantal squeezing properties of q-coherent states associated with a deformed harmonic oscillator and different quantum algebras (Buzek, 1991; Solomon and Katriel, 1993; McDermott and Solomon, 1994; Katriel and Solomon, 1990, 1991; Wang *et al.*, 1995; Wang and Kuang, 1992, 1993; Kuang and Wang, 1993; Si-Cong and Hong-Yi, 1995) have been studied in great detail and their applications to some concrete physical problems have been explored (Celeghini *et al.*, 1991, 1995; Chaichian *et al.*, 1990; Floratos, 1991).

The Pegg–Barnett s-dimensional truncated oscillator formalism (Pegg and Barnett, 1988, 1989; Barnett and Pegg, 1992; Gantsog *et al.*, 1992), which is a very important development in the field of quantum optics, moti-

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vated some authors to study the q-deformed harmonic oscillator and q-coherent states in finite-dimensional Hilbert space (Kuang, 1993; Yang and Yu, 1995), the relevance of quantum deformation in this truncated Fock basis being given by Ellinas (1992). For recent results on conventional coherent states in finite dimension see Buzek *et al.* (1992), Kuang *et al.* (1993, 1994), Miranowicz *et al.* (1994), Kuang and Chen (1994a, b), and Kuang and Zhu (1996).

The purpose of the present paper is to construct the even and odd q-coherent states of a harmonic oscillator in a finite-dimensional basis and study their properties. Then we investigate their three important optical statistics properties—namely, quadrature squeezing, antibunching, and amplitude-squared squeezing. Such an investigation reveals the role of the deformation parameter q on squeezing and may thus prove to be helpful in finding the physical meaning of the parameter, which until now has been unclear.

2. EVEN AND ODD q-COHERENT STATES AND THEIR PROPERTIES

We consider the $(s + 1)$-dimensional Hilbert space ψ, where s is an arbitrary positive integer. Then the number states $|n\rangle \in \psi$ are orthonormal and complete

$$\langle n|m\rangle = \delta_{n,m}, \qquad \sum_{n=0}^{s} |n\rangle\langle n| = 1 \tag{1}$$

The q-creation $a_q^\dagger$ and q-annihilation a_q operators and the number operator N are given by

$$a_q^\dagger = \sum_{n=1}^{s} \sqrt{[n]}|n\rangle\langle n-1|, \qquad a_q = \sum_{n=1}^{s} \sqrt{[n]}|n-1\rangle\langle n|, \qquad N = \sum_{n=0}^{s} [n]|n\rangle\langle n| \tag{2}$$

where $[x] = (q^x - q^{-x})/(q - q^{-1})$ and $0 < q < 1$, $1 < q < \infty$. The actions of the operators $a_q^\dagger$ and a_q are

$$\begin{aligned} a_q^\dagger|n\rangle &= \sqrt{[n+1]}|n+1\rangle; \qquad a_q^\dagger|s\rangle = 0 \\ a_q|n\rangle &= \sqrt{[n]}|n-1\rangle; \qquad a_q|0\rangle = 0 \end{aligned} \tag{3}$$

and the commutation relations are given by

$$a_q a_q^\dagger - q a_q^\dagger a_q = q^{-N} - [s+1]|s\rangle\langle s| \; [N, a_q] = -a_q, \qquad [N, a_q^\dagger] = a_q^\dagger \tag{4}$$

Now, we define the even and odd q-coherent states as

$$
\begin{aligned}
|z, s\rangle_q^{\text{even}} &= N^{\text{even}}(z, s)\cosh_q^s(za_q^\dagger)|0\rangle \\
&= N^{\text{even}}(z, s)\sum_{m=0}^{s/2}\frac{z^{2m}}{\sqrt{[2m]!}}|2m\rangle \\
|z, s\rangle_q^{\text{odd}} &= N^{\text{odd}}(z, s)\sinh_q^s(za_q^\dagger)|0\rangle \\
&= N^{\text{odd}}(z, s)\sum_{m=0}^{s/2-1}\frac{z^{2m+1}}{\sqrt{[2m+1]!}}|2m+1\rangle
\end{aligned}
\tag{5}
$$

where z is a complex number and the normalization constants are given by

$$
N^{\text{even}}(z, s) = \cosh_q^s(z\bar{z})^{-1/2}, \qquad N^{\text{odd}}(z, s) = \sinh_q^s(z\bar{z})^{-1/2} \tag{6}
$$

$\bar{z}$ is the complex conjugate of z and the two polynomial functions are

$$
\cosh_q^s(z\bar{z}) = \sum_{m=0}^{s/2}\frac{(z\bar{z})^{2m}}{[2m]!}, \qquad \sinh_q^s(z\bar{z}) = \sum_{m=0}^{s/2-1}\frac{(z\bar{z})^{2m+1}}{[2m+1]!} \tag{7}
$$

Here we have taken s = even = $2m$ (say), $m = 0, 1, 2, \ldots, s/2$. The results for s = odd will be given later. The orthogonality and completeness relations are respectively given by

$$
{}_q^i\langle z', s|z, s\rangle_q^i = N^i(z', s)N^i(z, s)\begin{cases}\cosh_q^s(z\bar{z}'), & i = \text{even}\\ \sinh_q^s(z\bar{z}'), & i = \text{odd}\end{cases} \tag{8}
$$

$$
{}_q^{\text{even}}\langle z', s|z, s\rangle_q^{\text{odd}} = 0
$$

and

$$
\begin{aligned}
I = \frac{1}{\pi}\int & \cosh_q^s(z\bar{z})|z, s\rangle_q^{\text{even}}\,{}_q^{\text{even}}\langle z, s| + \sinh_q^s(z\bar{z})|z, s\rangle_q^{\text{odd}}\,{}_q^{\text{odd}}\langle z, s| \\
& \times \exp_q(-|z|^2)\, d_q^2 z
\end{aligned}
\tag{9}
$$

Equations (8) and (9) show that even q-coherent states and odd q-coherent states are nonorthogonal, but they are orthogonal to each other and together they form a complete set.

With the help of equations (3) it can be easily checked that

$$
\begin{aligned}
a_q|z, s\rangle_q^{\text{even}} &= z\tanh_q^s(z\bar{z})^{1/2}|z, s\rangle_q^{\text{odd}} \\
a_q|z, s\rangle_q^{\text{odd}} &= z[\coth_q^s(z\bar{z})^{1/2}|z, s\rangle_q^{\text{even}} - \sinh_q^s(z\bar{z})^{-1/2}(z^s/\sqrt{[s]!})|s\rangle] \\
a_q^2|z, s\rangle_q^{\text{even}} &= z^2[|z, s\rangle_q^{\text{even}} - \cosh_q^s(z\bar{z})^{-1/2}(z^s/\sqrt{[s]!})|s\rangle] \\
a_q^2|z, s\rangle_q^{\text{odd}} &= z^2[|z, s\rangle_q^{\text{odd}} - \sinh_q^s(z\bar{z})^{-1/2}(z^{s-1}/\sqrt{[s-1]!})|s-1\rangle]
\end{aligned}
\tag{10}
$$

Equations (10) indicate that unlike the q-coherent states in infinite dimension (Wang *et al.*, 1995; Wang and Kuang, 1992 1993; Kuang and Wang, 1993; Si-Cong and Hong-Yi, 1995). The annihilation operator in the finite-dimension cannot act as a rotation between $|z, s\rangle_q^{\text{odd}}$ and $|z, s\rangle_q^{\text{even}}$. Moreover, $|z, s\rangle_q^{\text{even}}$ and $|z, s\rangle_q^{\text{odd}}$ are eigenstates of a_q^2 only in the limit $s \to \infty$, which is the well-known result for even and odd q-coherent states in infinite dimension (Wang *et al.*, 1995; Wang and Kuang, 1992, 1993; Si-Cong and Hong-Yi, 1995).

For s = odd = $(2m + 1)$ (say), $m = 0, 1, 2, \ldots, (s - 1)/2$, the even and odd q-coherent states are given respectively by

$$|z, s\rangle_q^{\text{even}} = \cosh_q^s(z\bar{z})^{-1/2} \sum_{m=0}^{(s-1)/2} \frac{z^{2m}}{\sqrt{[2m]!}} |2m\rangle$$

$$|z, q\rangle_q^{\text{odd}} = \sinh_q^s(z\bar{z})^{-1/2} \sum_{m=0}^{(s-1)/2} \frac{z^{2m+1}}{\sqrt{[2m+1]!}} |2m + 1\rangle \tag{11}$$

where

$$\cosh_q^s(z) = \sum_{m=0}^{(s-1)/2} \frac{z^{2m}}{[2m]!}, \qquad \sinh_q^s(z) = \sum_{m=0}^{(s-1)/2} \frac{z^{2m+1}}{[2m+1]!} \tag{12}$$

The orthogonality and completeness relations remain the same as in equations (8) and (9), respectively, with $\cosh_q^s(z)$ and $\sinh_q^s(z)$ given by equation (12). The actions of the operators a_q and a_q^2 on $|z, s\rangle_q^{\text{even}}$ and $|z, s\rangle_q^{\text{odd}}$ are given by the following relations:

$$\begin{aligned}
a_q|z, s\rangle_q^{\text{odd}} &= z \coth_q^s(z\bar{z})^{1/2}|z, s\rangle_q^{\text{even}} \\
a_q|z, s\rangle_q^{\text{even}} &= z[\tanh_q^s(z\bar{z})^{1/2}|z, s\rangle_q^{\text{odd}} - \cosh_q^s(z\bar{z})^{-1/2}(z^s/\sqrt{[s]!})|s\rangle] \\
a_q^2|z, s\rangle_q^{\text{odd}} &= z^2[|z, s\rangle_q^{\text{odd}} - \sinh_q^s(z\bar{z})^{-1/2}(z^s/\sqrt{[s]!})|s\rangle] \\
a_q^2|z, s\rangle_q^{\text{even}} &= z^2[|z, s\rangle_q^{\text{even}} - \cosh_q^s(z\bar{z})^{-1/2}(z^{s-1}/\sqrt{[s-1]!})|s - 1\rangle]
\end{aligned} \tag{13}$$

3. OPTICAL STATISTICS PROPERTIES OF EVEN AND ODD q-COHERENT STATES

3.1. Quadrature Squeezing

In order to study the q-squeezing property of $|z, s\rangle_q^{\text{even}}$ and $|z, s\rangle_q^{\text{odd}}$ we define Hermitian quadrature operators as (Ekert and Knight, 1989)

$$X_q^1 = \left(\frac{m\omega}{\hbar}\right)^{1/2} X_q^s, \qquad X_q^2 = (m\omega\hbar)^{1/2} P_q^s \tag{14}$$

with

$$X_q^s = \left(\frac{\hbar}{2m\omega}\right)^{1/2}(a_q^\dagger + a_q), \qquad P_q^s = i\left(\frac{1}{2}m\hbar\omega\right)^{1/2}(a_q^\dagger - a_q) \tag{15}$$

and they obey the uncertainty relation

$$\begin{aligned} &{}_q^i\langle z, s|(\Delta X_q^1)^2|z, s\rangle_q^i \; {}_q^i\langle z, s|(\Delta X_q^2)^2|z, s\rangle_q^i \\ &\geq \frac{1}{4}\,|{}_q^i\langle z, s|[X_q^1, X_q^2]|z, s\rangle_q^i|^2 \end{aligned} \tag{16}$$

where i = even, odd and the variances are defined by ${}_q\langle(\Delta X_q^j)^2\rangle_q = {}_q\langle X_q^{j2}\rangle_q - \langle X_q^j\rangle_q^2$, where $j = 1, 2$.

We will say a state q-squeezed in the X_q^j variable if

$${}_q^i\langle z, s|(\Delta X_q^j)^2|z, s\rangle_q^i\langle\tfrac{1}{2}\,|{}_q^i\langle z, s|[X_q^1, X_q^2]|z, s\rangle_q^i| \tag{17}$$

where $j = 1, 2$ and i = even, odd.

Making use of equations (3) and (5)–(7), we get the expectation values

$$\begin{aligned} &{}_q^{\text{even}}\langle z, s|(\Delta X_q^j)^2|z, s\rangle_q^{\text{even}} - \tfrac{1}{2}|{}_q^{\text{even}}\langle z, s|[X_q^1, X_q^2]|z, s\rangle_q^{\text{even}}| \\ &= r^2\left[\tanh_q^s(r^2) \pm \cos 2\theta\,\frac{\cosh_q^{s-2}(r^2)}{\cosh_q^s(r^2)}\right] \end{aligned} \tag{18}$$

and

$$\begin{aligned} &{}_q^{\text{odd}}\langle z, s|(\Delta X_q^j)^2|z, s\rangle_q^{\text{odd}} - \tfrac{1}{2}|{}_q^{\text{odd}}\langle z, s|[X_q^1, X_q^2]|z, s\rangle_q^{\text{odd}}| \\ &= r^2\left[\frac{\cosh_q^{s-2}(r^2)}{\sinh_q^s(r^2)} \pm \cos 2\theta\,\frac{\sinh_q^{s-2}(r^2)}{\sinh_q^s(r^2)}\right] \end{aligned} \tag{19}$$

where $j = 1, 2$ and $z = r\exp(i\theta)$.

Now equations (18) and (19) will respectively satisfy (17) provided

$$\begin{aligned} \cos 2\theta &\lessgtr \mp\frac{\sinh_q^s(r^2)}{\cosh_q^{s-2}(r^2)} \\ \cos 2\theta &\lessgtr \mp\coth_q^{s-2}(r^2) \end{aligned} \tag{20}$$

where the upper and lower signs correspond to $j = 1$ and 2, respectively.

Now from equations (7) it is evident that the right-hand sides of both the inequalities in (20) are positive. But, for $r^2 < 1$ and for all finite values of q, $\sinh_q^s(r^2)/\cosh_q^{s-2}(r^2)$ is less than 1 (which follows from the fact that $[n + 1]! > [n]!$). Therefore, the first inequalities in (20) are satisfied for $r^2 < 1$ and finite q, but the second inequalities are never satisfied.

Therefore for s = even, the even q-coherent states exhibit q-quadrature squeezing in both the variables X_q^1 and X_q^2 for $r^2 < 1$ and finite q, but the odd q-coherent states are not q-squeezed.

The degree of squeezing can be measured by the functions (Hong and Mandel, 1985a, b) S_A^{even} defined as ($A = X_q^1$ or X_q^2)

$$S_A^{\text{even}} = \frac{c}{d} \tag{21}$$

where

$$c = 2\,{}^{\text{even}}_{q}\langle z, s|(\Delta A)^2|z, s\rangle_q^{\text{even}} - |{}^{\text{even}}_{q}\langle z, s|[X_q^1, X_q^2]|z, s\rangle_q^{\text{even}}|$$

$$d = |{}^{\text{even}}_{q}\langle z, s|[X_q^1, X_q^2]|z, s\rangle_q^{\text{even}}| \tag{22}$$

The squeezing condition in this case takes the form

$$S_{X_q^1}^{\text{even}} < 0 \qquad \text{or} \qquad S_{X_q^2}^{\text{even}} < 0$$

Evaluation of the expectation values in equation (21) gives

$$S_A^{\text{even}} = \frac{e}{f} \tag{23}$$

where

$$e = 2r^2[\pm\cos 2\theta \cosh_q^{s-2}(r^2) + \sinh_q^s(r^2)]$$

$$f = qr^2 \sinh_q^{s-2}(r^2) + \cosh_q^{s-2}(q^{-1}r^2) - r^2 \sinh_q^s(r^2) \tag{24}$$

where the upper and lower signs correspond to $A = X_q^1$ and X_q^2, respectively.

In Fig. 1, $S_{X_q^1}^{\text{even}}$ is plotted for fixed s and q and for different values of $|z| = r$. From the figure it is evident that different values of the parameter

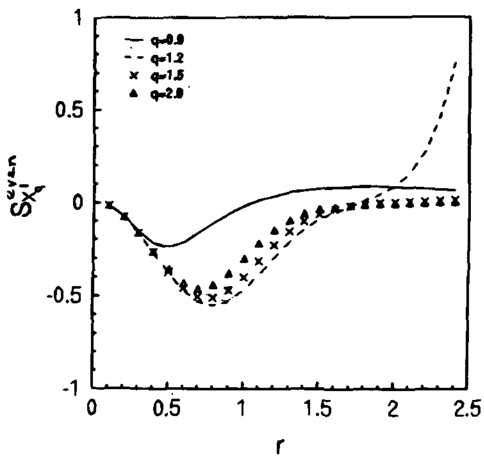

Fig. 1. Plot of $S_{X_q^1}^{\text{even}}$ for $s = 6$, $q = 0.9, 1.2, 1.5, 2.0$, and for different values of r.

q correspond to different squeezing, so that the deformation parameter q may be a quantum relevant to the degree of squeezing.

For the q-squeezing properties of $|z, s\rangle_q^{\text{odd}}$ and $|z, s\rangle_q^{\text{even}}$ for s = odd = $2m + 1$ (say), $m = 0, 1, 2, \ldots, (s - 1)/2$, we have

$$ {}^{\text{odd}}_{q}\langle z, s|(\Delta X_q^j)^2|z, s\rangle_q^{\text{odd}} \langle \tfrac{1}{2} | {}^{\text{odd}}_{q}\langle z, s|[X_q^1, X_q^2]|z, s\rangle_q^{\text{odd}} | \tag{25} $$

provided

$$ \cos 2\theta \lesseqgtr \mp \frac{\cosh_q^s(r^2)}{\sinh_q^{s-2}(r^2)} \tag{26} $$

and

$$ {}^{\text{even}}_{q}\langle z, s|(\Delta X_q^j)^2|z, s\rangle_q^{\text{even}} \langle \tfrac{1}{2} | {}^{\text{even}}_{q}\langle z, s|[X_q^1, X_q^2]|z, s\rangle_q^{\text{even}} | \tag{27} $$

provided

$$ \cos 2\theta \lesseqgtr \mp \tanh_q^{s-2}(r^2) \tag{28} $$

where in equations (26) and (28) the upper and lower signs correspond to $j = 1$ and 2, respectively.

The right-hand sides of equations (26) and (28) are both positive. But for $r^2 < 1$ and $0 < q < 1$, $1 < q < \infty$, $\cosh_q^s(r^2)/\sinh_q^s(r^2) > 1$, while $\tanh_q^{s-2}(r^2) < 1$ (this follows from the fact that $[n + 1]! > [n]!$). So conditions (28) are satisfied, but conditions (26) are not.

Therefore both for s = even and s = odd the even q-coherent states exhibit q-quadrature squeezing, but the odd q-coherent states do not.

3.2. Antibunching Effect

It is well known that if the normalized second-order correlation function of a light field (Walls, 1983) $g^2(0) < 1$, then the light field exhibits an antibunching effect. In a similar way, we introduce the second-order q-correlation function

$$ g_{q,i}^2(0) = \frac{{}^{i}_{q}\langle z, s|a_q^{\dagger 2} a_q^2|z, s\rangle_q^i}{{}^{i}_{q}\langle z, s|a_q^{\dagger} a_q|z, s\rangle_q^i}, \qquad i = \text{even, odd} \tag{29} $$

It is straightforward to evaluate the expectations in equation (29) and we have for s = even

$$ g_{q,\text{even}}^2(0) = \frac{\coth_q^s(r^2)\cosh_q^{s-2}(r^2)}{\sinh_q^s(r^2)} $$

$$ g_{q,\text{odd}}^2(0) = \frac{\tanh_q^{s-2}(r^2)\sinh_q^s(r^2)}{\cosh_q^{s-2}(r^2)} \tag{30} $$

From the discussions following equation (20) it can be proved that $|z$ $s\rangle_q^{\mathrm{odd}}$ exhibits an antibunching effect, but $|z, s\rangle_q^{\mathrm{even}}$ does not.

For s = odd we have

$$g^2_{q,\mathrm{odd}}(0) = \frac{\tanh_q^s(r^2)\sinh_q^{s-2}(r^2)}{\cosh_q^s(r^2)}$$

$$g^2_{q,\mathrm{even}}(0) = \frac{\coth_q^{s-2}(r^2)\cosh_q^s(r^2)}{\sinh_q^{s-2}(r^2)} \tag{31}$$

From the discussions following equations (26) and (28) it follows that $|z, s\rangle_q^{\mathrm{odd}}$ exhibits an antibunching effect, but $|z, s\rangle_q^{\mathrm{even}}$ does not.

So for both s = even and odd, the odd q-coherent states exhibit an antibunching effect, but the even q-coherent states do not.

3.3. Amplitude-Squared Squeezing

In analogy with the definition of amplitude-squared squeezing for the single mode of the electromagnetic field (Hillery, 1987), we introduce the q-analogue of the amplitude-squared squeezing in terms of q-quadrature operators

$$Y_q^1 = \frac{1}{2}(a_q^2 + a_q^{\dagger 2}), \qquad Y_q^2 = \frac{i}{2}(a_q^{\dagger 2} + a_q^2) \tag{32}$$

These operators obey the commutation relation

$$[Y_q^1, Y_q^2] = \frac{i}{2}(a_q^2 a_q^{\dagger 2} - a_q^{\dagger 2} a_q^2) \tag{33}$$

and the uncertainty relation

$$\begin{aligned} &{}_q^i\langle z, s|(\Delta Y_q^1)^2|z, s\rangle_q^i \, {}_q^i\langle z, s|(\Delta Y_q^2)^2|z, s\rangle_q^i \\ &\geq \tfrac{1}{4}|{}_q^i\langle z, s|[Y_q^1, Y_q^2]|z, s\rangle_q^i|^2 \end{aligned} \tag{34}$$

where i = even, odd and the variances are defined as usual.

A state is said to be q-amplitude-squared squeezed if

$${}_q^i\langle z, s|(\Delta Y_q^j)^2|z, s\rangle_q^i < \tfrac{1}{2}|{}_q^i\langle z, s|[Y_q^1, Y_q^2]|z, s\rangle_q^i| \tag{35}$$

where j = 1, 2 and i = even, odd.

Making use of equations (3) and (5)–(7) we get for s = even = $2m$ (say), $m = 0, 1, 2, \ldots, s/2$,

$$\begin{aligned} &{}^{\mathrm{even}}_q\langle z, s|(\Delta Y_q^j)^2|z, s\rangle^{\mathrm{even}}_q - \tfrac{1}{2}|{}^{\mathrm{even}}_q\langle z, s|[Y_q^1, Y_q^2]|z, s\rangle_q^{\mathrm{even}}| \\ &= \begin{cases} f_+(r, \theta, s) & \text{for} \quad j = 1 \\ f_-(r, \theta, s) - r^4x^2 & \text{for} \quad j = 2 \end{cases} \end{aligned} \tag{36}$$

where

$$f_{\pm}(r, \theta, s) = \frac{1}{2} \cosh_q^s(r^2)^{-1} r^4 \left\{ 2\cos^2 2\theta \left[\pm\cosh_q^{s-4}(r^2) \mp \frac{[\cosh_q^{s-2}(r^2)]^2}{\cosh_q^s(r^2)} \right] \right.$$

$$\left. \mp \cosh_q^{s-4}(r^2) + \cosh_q^{s-2}(r^2) \right\} \tag{37}$$

$z = r \exp(i\theta)$ and $x = \cosh_q^{s-2}(r^2)/\cosh_q^s(r^2)$.

Therefore

$$ {}^{\text{even}}_{\quad q}\langle z, s|(\Delta Y_q^j)^2|z, s\rangle_q^{\text{even}} < \tfrac{1}{2}|{}^{\text{even}}_{\quad q}\langle z, s|[Y_q^1, Y_q^2]|z, s\rangle_q^{\text{even}}| \tag{38}$$

when

$$\cos^2 2\theta < \begin{cases} \dfrac{y-1}{2A} & \text{for} \quad j = 1 \\ \dfrac{1}{2} - \dfrac{x-1}{2A} & \text{for} \quad j = 2 \end{cases} \tag{39}$$

where

$$y = \frac{\cosh_q^{s-4}(r^2)}{\cosh_q^{s-2}(r^2)} \qquad \text{and} \qquad A = y - x \tag{40}$$

Now it can be shown that $y < 1$ and $x < 1$ for all r and finite q. As $\cos^2 2\theta$ lies in the range (0, 1), the first of the inequalities in (39) will be satisfied when $A < 0$, but the second of the inequalities in (39) will be satisfied for $A > 0$, as well as for $A < 0$ and $\theta = (2n + 1)\pi/4$, $n = 0, 1, 2, \ldots$.

Therefore for s = even, both the components Y_q^1 and Y_q^2 are amplitude-squared squeezed for even q-coherent states.

For the odd q-coherent states we have

$$ {}^{\text{odd}}_{\ q}\langle z, s|(\Delta Y_q^j)^2|z, s\rangle_q^{\text{odd}} - \tfrac{1}{2}|{}^{\text{odd}}_{\ q}\langle z, s|[Y_q^1, Y_q^2]|z, s\rangle_q^{\text{odd}}|$$

$$= \begin{cases} g_+(r, \theta, s) & \text{for} \quad j = 1 \\ g_-(r, \theta, s) - r^4 t^2 & \text{for} \quad j = 2 \end{cases} \tag{41}$$

where

$$g_{\pm}(r, \theta, s) = \frac{1}{2} r^4 \sinh_q^s(r^2)^{-1} \left\{ 2\cos^2 2\theta \left[\pm\sinh_q^{s-4}(r^2) \mp \frac{\sinh_q^{s-2}(r^2)^2}{\sinh_q^s(r^2)} \right] \right.$$

$$\left. + \sinh_q^{s-2}(r^2) \mp \sinh_q^{s-4}(r^2) \right] \tag{42}$$

Therefore

$$
{}_q^{\mathrm{odd}}\langle z, s|(\Delta Y_q^j)^2|z, s\rangle_q^{\mathrm{odd}} < \tfrac{1}{2}|{}_q^{\mathrm{odd}}\langle z, s|[Y_q^1, Y_q^2]|z, s\rangle_q^{\mathrm{odd}}| \tag{43}
$$

when

$$
\cos^2 2\theta < \begin{cases} \dfrac{w-1}{2B} & \text{for } j = 1 \\[2ex] \dfrac{1}{2} - \dfrac{t-1}{2B} & \text{for } j = 2 \end{cases} \qquad t = \frac{\sinh_q^{s-2}(r^2)}{\sinh_q^s(r^2)} \tag{44}
$$

where

$$
B = w - t \quad \text{and} \quad w = \frac{\sinh_q^{s-4}(r^2)}{\sinh_q^{s-2}(r^2)} \tag{45}
$$

Now w and t are both less than 1 for all r and finite q. Therefore as $\cos^2 2\theta$ lies in the range (0, 1), the first of the inequalities in (44) will be satisfied when $A < 0$, but the second of the inequalities will be satisfied for $A > 0$ and for $A < 0$, $\theta = (2n + 1)\pi/4$, $n = 0, 1, 2, \ldots$.

Therefore the odd q-coherent states exhibit amplitude-squared squeezing for both the components Y_q^1 and Y_q^2.

For $s = \text{odd} = (2m + 1)$ (say), $m = 0, 1, 2, \ldots, (s - 1)/2$, equations (37)–(45) will remain the same as for s = even and consequently the results concerning the amplitude-squared squeezing for even and odd q-coherent states will also hold good.

Therefore both for s = even and s = odd the even and odd q-coherent states exhibit amplitude-squared squeezing for Y_q^1 and Y_q^2.

4. CONCLUSION

We have constructed the even and odd q-coherent states in a finite-dimensional Hilbert space and discussed some of their properties. It is found that though the properties of the finite-dimensional even and odd q-coherent states approach those of the even and odd q-coherent states in infinite dimension, these properties are nontrivially different if the dimension of the Hilbert space is finite. Also, there are significant differences in the aspects of the optical statistics properties of even and odd q-coherent states. For both s even and s odd, the even q-coherent states exhibit q-quadrature and q-amplitude-squared squeezing, while the odd q-coherent states show a q-antibunching effect and q-amplitude-squared squeezing for some values of z and for all finite q. Moreover, it is clear that different values of the parameter

q correspond to different degrees of squeezing, so that the deformation parameter may be a quantum relevant to the degree of squeezing.

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